MODERN TEACHING TECHNOLOGIES AND DEVELOPING CONSTRUCTIVE THINKING

Miroslava Mihajlov Carević[1]
Milena J. Petrović[2]
Nebojša Denić[3]

DOI: https://doi.org/10.31410/EMAN.S.P.2020.147

Abstract: *Numerous researches in the previous period have shown the effectiveness of mathematical learning with the use of information and communication technologies (ICT) and mathematical software packages. Also, through theoretical studies and research results, the quality of mathematics teaching in schools was examined. The results of the research of the authors of this paper showed the lack of a visual-logical approach in solving mathematical problems. Primary and secondary school students are primarily trained in mastering formulas and algebraic procedures that help them solve a given task. In order to develop the ability to perceive lawfulness and logical thinking, we organized introducing elementary and secondary school students to figurative numbers and selected examples that demonstrate the observation of lawfulness among numbers. We applied work in collaborative groups using computers and GeoGebra software. The results showed the students' ability to perceive lawfulness and to successfully solve tasks by applying the observed lawfulness. They also confirmed the effectiveness of mathematical learning with the use of computers, mathematical software and working in collaborative groups.*

Keywords: *Implementation, Visualization, Representation.*

1. INTRODUCTION

The twenty-first century is the era of widespread information and communication technologies. The new age requires technologically educated people, able to actively use modern technologies. Technological advances have also significantly influenced changes in the education system. The volume of teaching resources increased and additional demands placed on teachers (Bozkurt & Ruthven, 2016). Facing new challenges, contemporary teachers must be prepared to use new teaching aids in order to teach more effectively (Tabach, 2012). One of the main tasks of modern education is to introduce students to new technologies and to train them for their active use. A prerequisite for successful teaching and learning is the active role of all participants in the educational process (Doruk, Aktumen & Aytekin, 2013).

Hardware development has also contributed to the development of software packages such as GeoGebra, Cabri Geometry, Geometer's Sketchpad and others. Many researchers have been researching the effectiveness of mathematical learning by applying mathematical software packages (Hohenwarter & Fuchs, 2004; Hohenwarter, Hohenwarter & Lavicza, 2009; Lavicza & Varga, 2010; Mihajlov Carević, Kopanja & Denić, 2018; Takači, Stankov & Milanović, 2015). Research has shown that the integration of information and communication technologies into teaching processes makes teaching effective and interesting for students and also contributes to the development of students' active thinking by stimulating their creative thinking (Allegra,

1 Faculty of Mathematics and Computer Science, AlfaBK University, Belgrade, Serbia
2 Faculty of Science and Mathematics, University of Priština, Kosovska Mitrovica, Serbia
3 Faculty of Science and Mathematics, University of Priština, Kosovska Mitrovica, Serbia

Chifari & Ottaviano, 2001; Viamonte, 2010). In their study of the statistical evaluation of the realization of mathematics teaching using computers, the authors (Gavrilović, Denić, Petković, Zivić & Vujičić, 2018) considered 10 factors of influence. The main objective of the research is to analyze the factors that influence the quality of mathematics teaching related to the level of knowledge achieved during one school year as well as at the final exam. An adaptive neuro fuzzy inference system (ANFIS) software package was used to determine the qualitative impact of 10 factors on the performance of mathematics. The survey found that the percentage of students studying with educational software had the greatest influence on the average score on the final exam. Combining 2 factors, the authors of this study came to the conclusion that the results at the final exam would be higher if the classes taught were interesting to the students with the learning through educational software. Combining the three factors, the aforementioned 2 factors have the strongest influence on the grade point average on the final exam, with the factor containing a better grade point average on the initial math test.

The authors of this paper conducted several studies with primary and secondary school students. All research has confirmed the efficiency and interest of mathematics learning using computers and software packages. The students are undoubtedly interested and very much in the mood for innovation in teaching. New approaches to presenting matter in class are welcome. In this context, modern technologies in the education system are of particular importance and contribution. In addition to these positive observations, the authors of this paper stated that primary and secondary school students are not sufficiently well-versed in the visual-logical approach to solving mathematical problems. When asked to elementary school students what is the sum of odd numbers less than 100, a small number of correct answers were obtained. Only 15.8% of the students answered the question correctly. It was noted that most students are trying to come up with a solution by summing all the numbers 1 + 3 + 5 +… . Research with sixth grade students is described in the paper „Numbers to develop a logical approach to solving tasks with numerous strings"(Mihajlov Carević, Petrović & Denić, 2018). Similar results were obtained in research with seventh grade elementary school students (Mihajlov Carević, Kopanja & Denić, 2017). Eighth grade students are not covered by the research because of their involvement in the preparation of the final exam at the end of primary education. The authors of this paper chose the first grade of „gymnasium" for research in high school. When asked what the sum of the first 1000 natural numbers is, only 12.8% of students gave the correct answer. The complete results of this research are presented in the paper „Figurative numbers contribution to the perception of legality in numerous strings tasks and long-term memory of numerous data" (Mihajlov Carević, Petrović & Denić, 2019). The obtained results showed that elementary and secondary school students were not taught to observe lawfulness and to solve problems by applying the observed lawfulness. Therefore, a survey was conducted in which students were introduced to selected examples demonstrating the recognition of lawfulness among numbers.

2. RESEARCH METHODOLOGY

In order to compare the results at the end of the survey, two groups of students were formed in each survey, experimental and control. Both groups are composed of approximately the same number of students with approximately the same grade point average in mathematics. Both groups organized student collaborative work by forming small collaborative groups. Working in collaborative groups has been recommended by many researchers as an extremely effective approach to learning (Dooly, 2008; Chai, Lin, So & Cheah, 2011; Petrović & Kontrec, 2017). Examples were selected to demonstrate the identification of regularities among numbers and examples were prepared with figurative numbers used in the experimental groups. Figurative numbers with their pictorial representations and the laws that apply to their members are very interesting, easy to understand for students and can be a very good tool for presenting paradigms and developing students' constructive thinking (Mihajlov Carević, Kopanja & Denić, 2017).

The tasks of the research are to answer the following research questions:

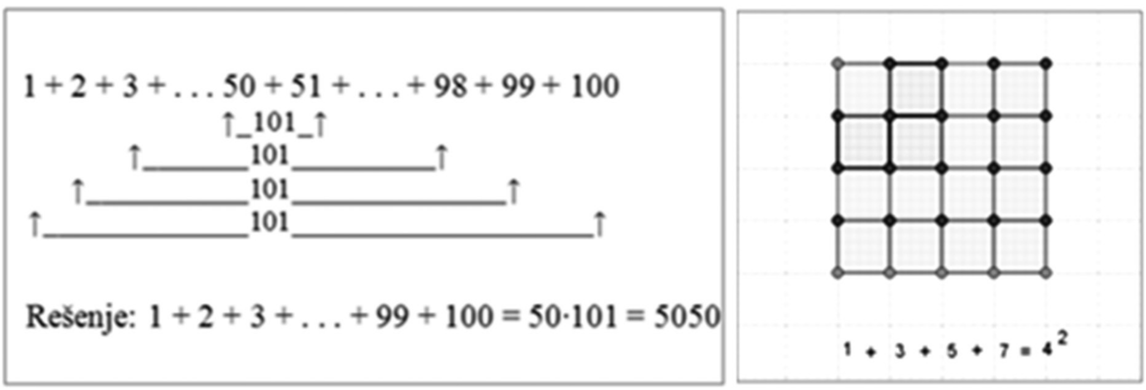
1. Are elementary (middle) high school students able to recognize the regularities among numbers when solving problems with numerous arrays and sets?
2. Do selected examples for demonstrating lawfulness among numbers contribute to developing lawfulness ability?
3. Does working with figurative numbers contribute to recognizing the lawfulness of numbers and developing constructive thinking?

The hypotheses in the research are as follows:

1. It is assumed that less than 25% of students will be able to complete tasks requiring the identification of lawfulness among numbers.
2. Well-chosen examples for demonstrating lawfulness among numbers will contribute to developing the ability to observe lawfulness and solve problems with numerous arrays and sets.
3. Working with figurative numbers will help students solve tasks by applying the observed legality.

At the beginning of each survey, the students solved the initial test, which was to test the students' learning to perceive the regularities among the numbers and to apply the observed regularities to solve the task. The results of the initial test, in all studies, confirmed the authors' first hypothesis. In all groups, less than 25% of students completed the tasks correctly. After that, students were introduced, in all groups, to selected examples to demonstrate the recognition of lawfulness among numbers as well as figurative numbers.

Among the examples selected were the Gaussian procedure for calculating the sum of the first 100 natural numbers (Figure 1_A) and the Pythagoras method for calculating the sum of odd numbers (Figure 1_B).



A B

Figure 1. Gaussian procedure for calculating the sum and displaying the sum of odd numbers

Figurative numbers were used in the experimental groups and are represented by triangular and square numbers (in the sixth grade of primary school), pentagonal and hexagonal numbers (in the other groups). Students are introduced to the graphical representation of triangular numbers (Figure 2), then to the laws that apply to the differences between two adjacent figurative numbers and the differences between the previous differences.

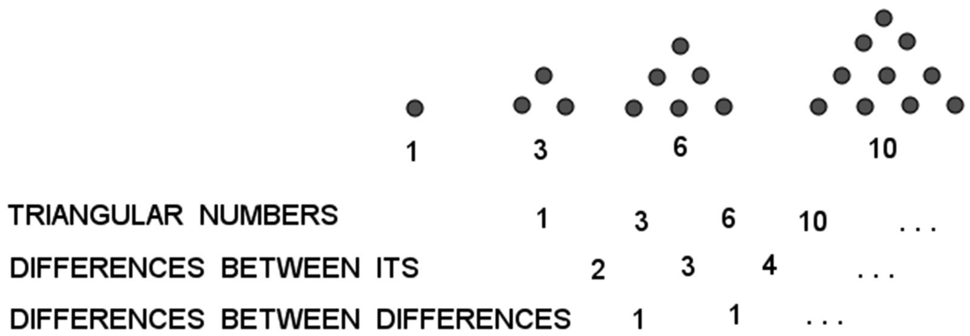


Figure 2. Showing triangular numbers with differences

Then, students were instructed to determine the given triangle number in the series by applying the observed differences between the triangular numbers (Figure 3). The same was done with other figurative numbers.

second triangular number: $3 = 1 + 2$
third : $6 = 1 + 2 + 3$
fourth : $10 = 1 + 2 + 3 + 4$
fifth : $15 = 1 + 2 + 3 + 4 + 5$
. . .
thirtieth : $x = 1 + 2 + 3 + 4 + 5 + \dots + 30$

$$x = 1 + 2 + 3 + \dots + 15 + 16 + \dots + 28 + 29 + 30$$

↑_31_↑
↑_________31_________↑
↑_____________31_____________↑
↑_________________31_________________↑

It is $x = 15 \cdot 31 = 465$

Figure 3. Procedure for determining the thirtieth triangular number

Upon completion of the planned work (in all studies), both groups, experimental and control, performed a test to verify acquired ability to detect regularities among numbers and to apply observed regularities in solving tasks. The results obtained are summarized and statistically processed. The test results, in all studies, were significantly better than the results of the initial test. This proves the second and third hypotheses in the research that well-chosen examples for demonstrating lawfulness among numbers will contribute to developing the ability to detect lawfulness and solve problems with numerous arrays and sets. Figure 4 shows a graphical comparison of the results of the experimental and control group of sixth grade students.

By comparing the results of the experimental group that worked with the figurative numbers and the control group, it was concluded that the results of the experimental group (in all studies) were better than the results of the control group.

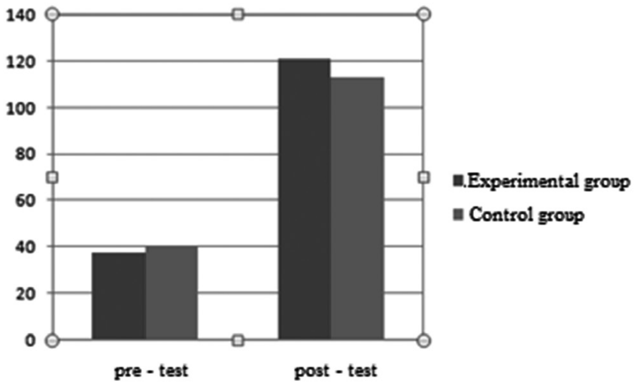


Figure 4. Results on the pre-test and post-test in the experimental and control group

After realizing the planned exercise of a visual-logical approach to solving problems with numerous arrays and sets, both groups, experimental and control (in all studies) made progress. This fact shows that well-chosen examples guide students in visualizing solutions to problems. It also shows that figurative numbers can be an instrument for instructing and practicing students to detect lawfulness among numbers and to solve problems by applying observed lawfulness.

3. FUTURE RESEARCH DIRECTIONS

On the basis of everything presented here, we will continue to research teaching aids to develop students' constructive thinking.

4. CONCLUSION

The realisation of mathematics teaching in the modern age requires the implementation of ICT and educational software. The research of the authors of this paper shows that, in addition, more attention should be paid to the visual-logical approach of solving mathematical problems. Visualization and representation are extremely important and useful in the process of learning and understanding mathematics (Duval, 1999; Arcavi, 2003; Van Garderen & Montague, 2003; Wang, Wu, Kinshuk & Spector, 2013). Researcher results (Gavrilović, Denić, Petković, Živić & Vujičić, 2018) confirm that educational software applications could produce the best results in a math class. Future research should focus on examining educational software that produces a visual-logical approach to solving mathematical problems.

REFERENCES

Allegra M., Chifari A. & Ottaviano S. (2001). ICT to train students towards creative thinking, *Educational Technology & Society,* 4(2), 48-53.

Arcavi, A. (2003). The role of visual representations in the learning of mathematics.
Educational studies in mathematics, 52(3), 215-241.

Bozkurt, G. & Ruthven, K., (2016). Classroom-based professional expertise. A mathematics teacher's practice with technology, *Educ. Stud. Math*., DOI 10. 1007/s 10649-016-973-5.

Chai, C., Lin, W.-Y., So, H.-J. & Cheah, H.M. (2011). *Advancing collaborative learning with ICT: Conception, cases and design.* Singapore: Ministry of Education.

Dooly, M. (2008). *Constructing knowledge together.* In M. Dooly (Ed.), Telecollaborative language learning. A guidebook to moderating intercultural collaboration online (pp.21-24). Bern: Peter Lang.

Doruk, B. K., Aktumen, M. & Aytekin, C. (2013). Pre-service elementary mathematics teachers' opinions about using GeoGebra in mathematics education with reference to 'teaching practices'. *Teaching Mathematics and Its Applications*, 32(3), 140-157.

Duval, R. (1999). Representation, Vision and Visualization: Cognitive Functions in Mathematical Thinking. *Basic Issues for Learning.*

Gavrilović, S., Denić, N., Petković, D., Živić, N. V., & Vujičić, S. (2018). Statistical evaluation of mathematics lecture performances by soft computing approach. *Computer Applications in Engineering Education*, *26*(4), 902-905.

Hohenwarter, M. & Fuchs, K. (2004). Combination of dynamic geometry, algebra and calculus in the software system GeoGebra. In *Computer Algebra Systems and Dynamic Geometry Systems in Mathematics Teaching Conference.*

Hohenwarter, J., Hohenwarter, M. & Lavicza, Z. (2009). Introducing dynamic mathematics software to secondary school teachers: The case of GeoGebra *Journal of Computers in Mathematics and Science Teaching*, *28*(2), 135-146.

Lavicza Z. & Varga Z. P. (2010). Integrating GeoGebra into IWB-equipped teaching environments: preliminary results, *Technology, Pedagogy and Education,* Vol. 19, No.2, 245-252.

Mihajlov Carević, M., Kopanja, L. & Denić, N. (2017). Figurativni brojevi kao sredstvo za prezentaciju paradigmi i razvijanje konstruktivnog mišljenja, *Nacionalna konferencija sa međunarodnim učešćem, Čačak, Zbornik radova ITOP17,* 217-224.

Mihajlov Carević, M., Kopanja, L. & Denić, N. (2018). GeoGebra as a tool for understanding and learning of mathematical quantities in Cartesian coordinate system, *The first international conference on education MICE 2017, "Promjena stvarnosti kroz obrazovanje", Mostar*, ISSN: 2303-7342, str. 221-226, 2018.

Mihajlov Carević, M., Petrović, M. & Denić, N. (2018). Numbers to development of visual logical approach to solving tasks with numerous strings, *Međunarodna naučna konferencija „Savremene perspektive vaspitno-obrazovnog rada", Internacionalni univerzitet u Novom Pazaru, Univerzitetska misao – časopis za nauku, kulturu i umjetnost,* (17), 72-85.

Mihajlov Carević, M., Petrović, M. & Denić, N. (2019). Figurative numbers contribution in perceiving the legality in numerous strings tasks and long-term memory of numerous data, *EURASIA Journal of Mathematics, Science and Technology Education,* 15(4): em1692.

Petrović, M. & Kontrec, N. (2017). Possibilities for applying team teaching system in order to improve the efficiency of math teaching, *A collection of papers from the National Conference with international participation, Leposavić: Innovation in education- digitalization, innovation models and programs.*

Tabach, M. (2012). A mathematics teacher's practice in a technological environment: A case study analysis using two complementary theories, *Technology, Knowledge and Learning,* 16(3), 247-265.

Takači, Đ., Stankov, G. & Milanović, I. (2015). Efficiency of learning environment using GeoGebra when calculus contents are learned in collaborative groups, *Elsevier Ltd,* 0360-1315.

Van Garden D., Montague M., (2003). Visual-spatial representation, mathematical problem solving and students of varying abilities*, Learning Disabilities Research & Practice,* 18(4), 246-254.

Viamonte, A. J. (2010). The computer in the teaching of mathematics. In Proceeding of advanced educational technologies. *6th WSEAS/IASME International conference on educational technology (EDUTE'10) (pp.* 24-29).

Wang, M., Wu B., Kinshuk, Chen N. S., Spector J. M., (2013). Connecting problem-solving and knowledge-construction processes in a visualization-based learning environment, *Computers & Education*, 68, 293-306.