IMPACT OF NON-LINEAR VOLATILITY IN STOCK-SPECIFIC RISK ON THE TURNOVER OF ACTIVELY MANAGED PORTFOLIOS

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Abstract: *Active investment has been established as one attractive approach for portfolio management. In order to achieve additional return – alpha, it requires investors to rebalance their portfolios often and to apply it for broader set of assets. However, as a result of such strategy portfolios could be exposed to an enormous turnover which leads to higher transaction costs. In many cases models with proven high-quality fail to provide the projected alpha because of alpha decaying caused by transaction costs of high turnover.*

Our paper is aimed to give more details about influence of stock-specific risk on turnover of active investments. We find that the ratio between target tracking error of the portfolio and stock-specific risk of an important factor in establishing the optimum turnover (and transaction costs). We investigate how this ratio is related with the turnover and how it influences the portfolio optimization process. We prove that changes in stock-specific risk causes managers to rebalance their portfolios in order to achieve their target tracking error. It is shown that these changes occur due to the non-linearity of stock volatility. We use GARCH model to measure the impact of short-term volatility shocks on the turnover of portfolio. Our findings confirm the importance of non-linear volatility for active portfolio turnover. Furthermore, we present empirical example for keeping turnover in desired level by adjusting the target tracking error of the factor portfolio.

Keywords: *turnover, non-linear risk, transaction costs, alpha.*

1. INTRODUCTION

Active portfolio management is a well-established approach for making investment decisions in portfolio theory and practice. Its logic can be described by this simple formula developed by Grinold in [1].

$$E(r_i) = \sigma_{r_i}\ IC\ \ z_i \tag{1}$$

where

$E(r_i)$ is the expected additional active return - alpha;
σ_{ri} – individual stock specific risk;
IC – information coefficient
z_i – standardized risk-adjusted scorings;

(1) presents the main idea of active management – higher expected return can be achieved if the stock diverts enough from the benchmark (σ_{ri}), if the forecasting factor gives enough explanatory power (IC) and if the factor has positive value for that stock resulting in higher score for it (z_i). Each of these 3 indicators is variable during the time t for every stock i in portfolio. This means

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that whenever some of the three factor changes its values new opportunity for alpha appears. To benefit from this opportunity, portfolio managers should rebalance their portfolio according to the factor's signal. That gives specific feature of the active management – to be active. Managers constantly must rebalance their portfolio by increasing/decreasing the weights in those stocks which experience positive/negative changes in indicators from (1) which means to buy or sell some stocks. This activity causes turnover and transaction costs. Therefore, investors meet enormous collision in objectives of their management. From one side, to maximize alpha they must use every opportunity for alpha by changes the weights of assets whenever factors from (1) change; from other side, because alpha can be decaying from transaction costs management should minimize changes in the weights.

In this paper we concentrate our attention on the problem with transaction costs of active portfolio management caused by high turnover. We verify the previous theoretical findings that portfolio turnover is dependent on the autocorrelation of the forecasting factor. Furthermore, we find that non-linear volatility in stock-specific risk is correlated with the active portfolio turnover. Incorporating our findings, we empirically test how to control the ratio between target tracking error and stock-specific risk in order to maintain desired level of portfolio turnover.

2. THEORETICAL REVIEW

Qian, Sorensen and Hua (QSH) in [2] prove that transaction costs should be directly integrated into alpha optimization modeling as an indigenous factor. They apply unconstrained mean-variance optimization for active portfolio. Assuming that all stock specific active risks are constant and the number of stocks is unchanged, they present that the optimal active weight of each stock in the portfolio should follow the pattern of (2)[3].

$$w_i^* = \frac{z_i}{\sqrt{N-1}} * \frac{\sigma_{TE}}{\sigma_{r_i}} \tag{2}$$

where:

w^*_i is the optimum active weight of asset i in portfolio;
σ_{TE} – targeted active risk (tracking error) of the portfolio;
N – number of stocks in portfolio.

In [2] QSH concentrated their interest on the correlation between forecasts and . Their main contribution is the proof that the portfolio turnover is an algebraic function of one-leg autocorrelation in forecasts and as such it is an important diagnostic for evaluating factors. However, in their paper QSH assume that the stock-specific risk is constant. They even assume that this risk is the same among the all stocks. By this way they exclude from their analysis the changes in stock-specific risk as a factor influencing turnover. However, (2) is fundamental for our research. Here, for the first time we observe an important relationship – the optimum active weight depends on the ratio between target active risk for the portfolio and stock specific risk. This ratio has strong economic meaning – it presents the ratio between what the managers target as an active risk and the quality of what they have as an available material (risk of the stocks) to build their portfolio - .

[3] In (2) we changed the places of and without changing the logic of original QSH's formula. We also use risk-adjusted scorings instead of originally introduced risk-adjusted forecasts. This substitution doesn't change the results and conclusions.

During the process of developing the theory of active portfolio management authors usually assume constant IC. First Qian and Hua in [3] and later Ye in [4] introduced the idea of the volatility of IC - . The final and most general explanation of this risk has been given by Ding and Martin in [5]. They argue that the total active portfolio risk involves three parts: (1) stock-specific risk of the asset, involved in the portfolio, factor risk, presented by , (2) factor risk presented by and (3) strategy risk presented by the dispersion of the errors in cross-section regression of forecast - . In our previous study [6] we prove that Ding and Martin's variant measures total active risk more accurately and therefore must be always taken into consideration in active portfolio management. This is especially valid for active management turnover.

Ding, Martin and Yang (DMY) in [7] develop QSH's model involving in it another risks - factor risk and strategy risk. We present the formula - (3) slightly changed from its original form by rewriting in the same manner as in (2).

$$w_i^* = \frac{z_i}{N\sqrt{\frac{1-\mu_{IC}^2-\sigma_{IC}^2}{N}+\sigma_{IC}^2}} * \frac{\sigma_{TE}}{\sigma_{r_i}} \tag{3}$$

where

μ_{IC} is the average time series IC of the model.

In (3) the two new elements of active risk are involved in the enumerator of formula for optimal stock weights and present there with element . DMY explained that this formula is in fact more general variant of (2). If we assume the only difference between (2) and (3) is that in the latter presents N instead of N-1. Obviously, DMY develop with (3) more general explanation of factors influencing the optimal weights of assets in active managed portfolio. Additional to previous 4 factors, described by QSH – N, , and , here we see another important factor - . The larger the factor risk is the lower the weight of this stock in the portfolio is. Logic behind this relationship is that whenever there is uncertainty in the forecast results, managers try to stay closer to benchmark weights. In [7] DMY scrutinize farther this relationship. However, they again do not go deeply in the role of for portfolio turnover.

For us (3) again, as in (2), shows importance of the ratio. We try to investigate what is the influence of this ratio on the turnover of actively managed portfolio. To do so we have to observe how the weights are changing with changes in . Every change in weghts leads to transaction costs. Therefore, if we observe changes in weights caused by the changes in stock-specific risk, this will be serious sign of increasing the turnover and transaction costs.

3. METHODOLOGY

To find the role of risk changes we allow in (3) the stock-specific risk to change and all the factors stayed the same. We can rewrite (3) into (4).

$$| w_{i(t)}^* - w_{i(t-1)}^* | = \frac{z_{it}}{\sqrt{\frac{1-\mu_{IC}^2-\sigma_{IC}^2}{N}+\sigma_{IC}^2}} * \frac{\sigma_{TE}}{|\sigma_{r_{i(t)}} - \sigma_{r_{i(t-1)}}|} \tag{4}$$

Formula (4) is fundamental for our research. It shows that every change in individual will result in changes in the weights urging portfolio managers to sell or buy that stock and this will cause transaction costs.

As a first step of our research we calculate the turnover for our hypothetical portfolio on TWSE with our selected factors for forecast[4]. We apply (4) to find what is the monthly turnover according to DMY's model – column 6 of Table 1. First, we observe serious differences in the turnover between the three groups of factors: while the fundamental factors require turnover between 7.2% up to 28.2% (for TO and ROE), the technical factors require turnover between 160.1% and 201,1%. This result is according the theory developed in [2] which stays that factors with lower autocorrelation will produce high turnover – because such factors change often in (4).

Factor group	Factor	IC	σ_{IC}	Factor Autocorrelation	QHS Turnover	DMY Turnover
	(column 1)	(column 2)	(column 3)	(column 4)	(column 5)	(column 6)
Fundamental factors	FF-EBA	0.073	0.253	0.997	16.4%	8.5%
	FF-NM	0.057	0.205	0.993	25.5%	15.2%
	FF-OM	0.046	0.218	0.996	20.1%	11.5%
	FF-ROA	0.083	0.243	0.995	21.5%	11.4%
	FF-TO	0.017	0.268	0.998	14.6%	7.2%
	FF-ROE	0.103	0.197	0.978	45.9%	28.2%
Technical factors	TF-20DM	0.033	0.172	0.042	304.1%	201.7%
	TF-30DM	0.035	0.193	0.294	261.1%	162.1%
	TF-BOL	-0.030	0.184	0.096	295.4%	188.4%
	TF-MA	0.029	0.176	0.069	299.8%	196.3%
	TF-PP	0.025	0.194	0.305	259.0%	160.1%
Market factors	MF-BP	-0.028	0.182	0.990	31.4%	20.1%
	MF-EP	0.092	0.195	0.958	63.8%	39.3%
	MF-SP	-0.014	0.199	0.995	22.4%	13.6%

Table 1: Turnover for TSE Portfolio

We compare this turnover with the turnover calculated by QSH's model[5] – presented in column 5. The QSH's turnover is dramatically higher than those from DMY model. This is result of involvement of factors risk in (4). Because there is higher uncertainty in forecasts, managers do not change aggressively the weights of the stocks. This result confirms the advantages of DMY model because it gives results more closely to applied in investment practice turnover, and in our paper, we follow it.

4. NON-LINEARITY ON STOCK-SPECIFIC RISK AND ITS MANAGEMENT

According to (4) additional turnover arises when investors change their expectation for stock-specific risk from to . In portfolio practice a difference in expected stock-specific risk appears and it is due to the non-linear volatility. This phenomenon is well-researched topic in empirical finance. Most common approach to examine non-linear volatility are GARCH models, initially developed by Bollerslev in [8]. Using the GARCH (1,1) model we get the change in expected risk – (5)[6]:

$$\Delta\sigma_{t+1}^2 = \alpha * (\epsilon_t^2 - \epsilon_{t-1}^2) + \beta * (\sigma_t^2 - \sigma_{t-1}^2) \tag{5}$$

4 For the portfolio and factors see Appendix 1.

5 The turnover according QSH' model (2) has been calculated in the same manner as (4) for (3) - $|\, w_{i(t)}^* - w_{i(t-1)}^*| = \frac{z_{it}}{\sqrt{N-1}} * \frac{\sigma_{TE}}{|\sigma_{r_{i(t)}} - \sigma_{r_{i(t-1)}}|}$

6 The detailed derivation is given in Appendix2.

We prove this GARCH effect on the turnover with our factor portfolios. Figure 1 shows the correlation between each stock's turnover within the factor portfolios and their GARCH -parameters.

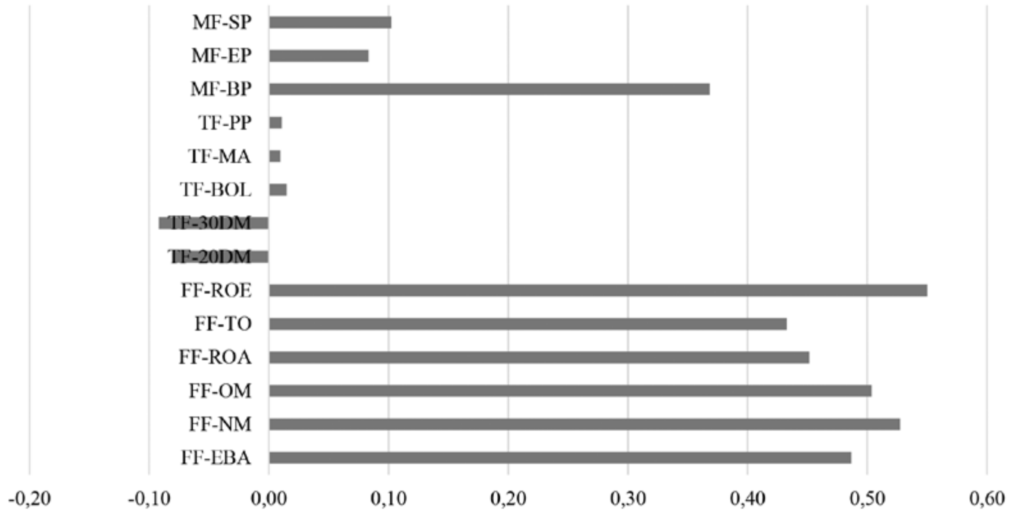

Figure 1. Correlation between stock turnover and -parameters from GARCH by different factor portfolios

Evidently, there is a strong positive correlation between the -parameter of the GARCH equation and the turnover of the stock. This means that stocks with higher sensitiveness to volatility shocks tend to require higher turnover within the factor portfolio. The only exceptions are the technical factor portfolios where there is no correlation between the two variables. This is explained by the nature of these factors[7]. Whenever there are volatility shocks on the market this requires managers to change the weights of actively managed assets in order to keep their goals. Therefore, the practice of assuming linearity in stock-specific return is not correct and can lead to higher than expected turnover which will decay active returns.

According to DMY investors can control the active portfolio turnover by changing the target tracking error. In essence, the is a parameter that governs the amount of additional risk that the active portfolio takes. In equation (4) there is very useful ratio between and the average expected stock-specific risk. Keeping this ratio constant, according to (4) will result in no-additional turnover due to non-linear volatility. Therefore, when we relax the assumption of linear stock-specific risk then investors must correct their target tracking error to compensate for the change or must bear the transaction costs on this additional turnover. For example, if volatility shock happens then for the next period it is necessary to raise the target portfolio risk to sustain the desired level of turnover and trading costs. Oppositely, if there is lower expected stock-specific risk then investors need to cut the .

In our investment universe the stock-specific risk is declining. This means that to sustain the turnover at certain level the target also must be decreased. This approach to portfolio construction process is very intuitive, because the mean-variance optimization can produce optimal portfolio for desired alpha return. Then by controlling turnover with the help of investor can decide what portion of the expected alpha return to be spend as transaction costs.

7 Technical factors rely on forecasting price movements with only market information and when volatility shock happens then usually technical factors change their behavior. However, this is only one possible explanation, thus this issue must be studied further in future research.

Next, we show the estimated target portfolio for each of the factor portfolios. The settings assume that half of the expected alpha to be sacrificed as transaction costs. For comparison purposes we optimize each factor portfolio to have 3.66% expected annual alpha and this suggests desired turnover level of 30% in each month (rebalancing period). The results are presented in Figure 2.

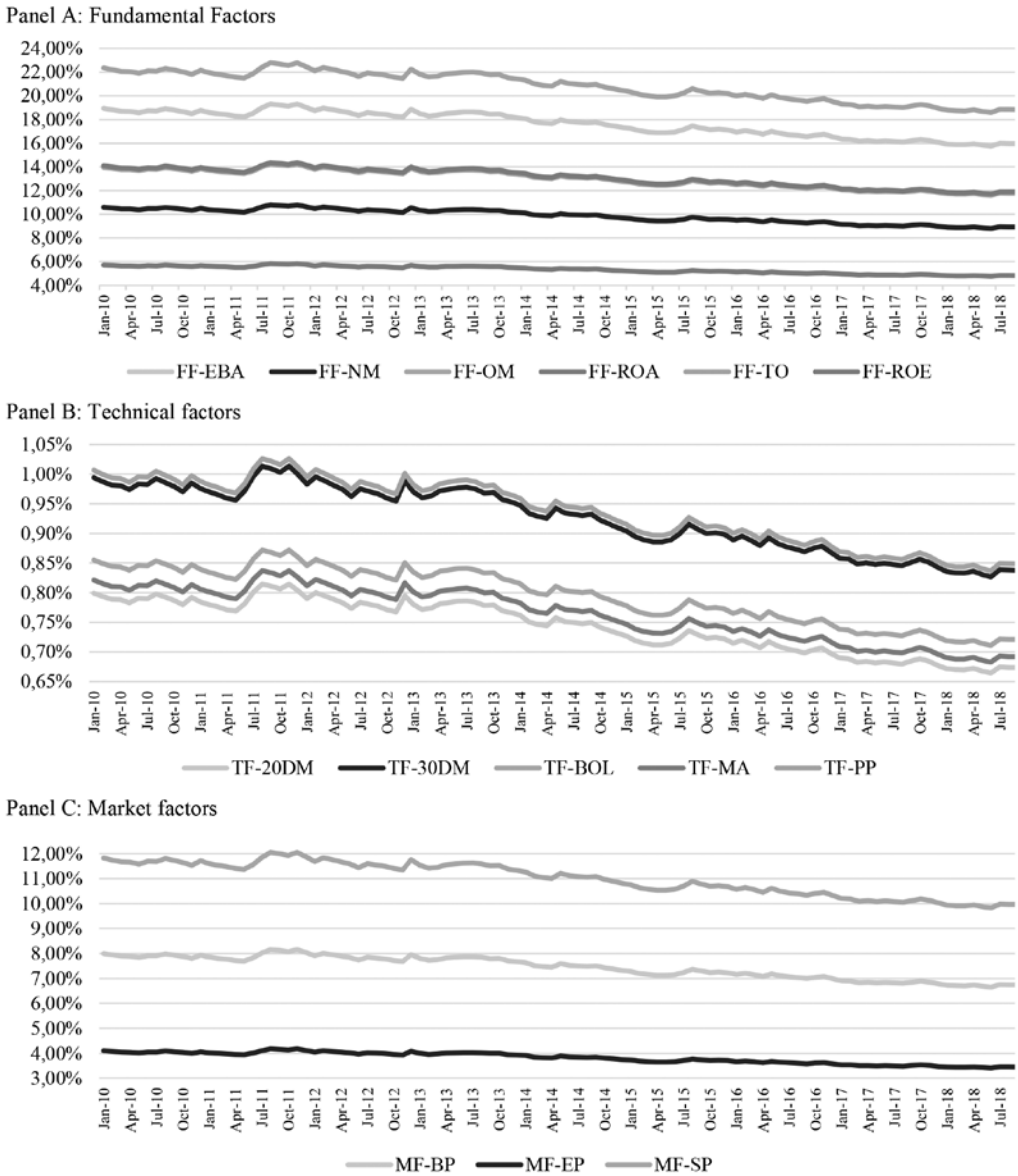

Figure 2. Changes in required to keep constant turnover of each portfolio at 30%

Following the decrease in stock-specific risk, there should be corresponding lowering of the across factors in order to keep constant. For example, for FF-OM managers must shift the from 10.3% to 12.4% in order to keep 30% turnover. The most volatile change is in the technical factors because their inherently higher turnover makes the impact of non-linear volatility bigger. In absolute terms the fundamental factors require higher change in . It is because these portfolios have less strategic risk. This allows managers of such portfolios to target higher levels . As a result, the impact of non-linear volatility is higher in absolute terms. The average standard deviation compared to the mean of the is just over 6%. This means that to sustain the desired turnover investors must shifts in by an average of 6%. This subtle change does not seem a lot, however if the incurred transaction costs are accumulated across the full periods they can “eat” big portion of the alpha return.

5. CONCLUSION

We analyze the models given by QSH and DMY for estimating the turnover. DNY's model provides more precise evaluation of turnover because it takes into consideration factor risk. We prove that this cases a sufficient difference between results of the two models.

We prove that the popular investment practice of assuming linearity in stock-specific risk is not correct and can lead to higher than expected turnover which will cause decaying of active returns. In order to be more precise in establishing the turnover the GARCH effect of stock-specific risk should be involved in portfolio risk models. For our stock sample we observe strong positive correlation between the -parameter of the GARCH equation and the turnover of the stocks. Stocks with higher sensitiveness to volatility shocks tend to require higher turnover and therefore this will case deeper alpha decaying for that portfolios.

We suggest that in order to manage the volume of turnover portfolio managers must keep ratio constant. This means that whenever because of non-linearity the stock-specific risk changes they have to provide opposite changes in . In our stock sample managers must change their target with about 6%. Our result can help portfolio managers to adjust their strategy in order to prevent alpha decaying in their portfolios.

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APPENDIX 1:
Market, Index and Factors Selected for Tests

For testing the non-linear volatility of stock-specific risk over active management we select Taiwan stock Exchange (TWSE). TWSE has been chosen because its market characteristics – efficient enough which makes it part of developed markets but with high volatility which is more typical for emerging markets in Southeast Asia. This makes it perfect for testing the influence of diverse factors over diverse type of stocks.

As a benchmark of our portfolio the index TSEC 50 has been chosen. We chose this index because it is not too broad from one side but involves enough variety of stocks to be forecasted. As some of the stocks do not fulfil criteria to be involved in the portfolio (like fundamental and market data availability) we exclude 6 of the stock resulting with 44 stocks in our benchmark portfolio. We assume that this 44 – asset portfolio will be managed actively according (4). The weight of one stock increases from the weight of the same stock in the benchmark if: (a) the score for it according the signal from the factor increases, (b) the target portfolio risk increases, (c) the autocorrelation in the factor decreases and (d) factor risk decreases. Number of stocks is assumed constant – 44. Our sample period is January, 2010 to September, 2018. Rebalancing of the portfolio is done every month.

Factor group	Factor Symbol	Factor Name	Source of information
Fundamental factors	FF-EBA	EBITDA to Assets	Income statement and Balance sheet
	FF-NM	Net Margin	Income statement
	FF-OM	Operating Margin	Income statement
	FF-ROA	Return-on -Asset	Income statement and Balance sheet
	FF-TO	Total Asset Turnover	Income statement and Balance sheet
	FF-ROE	Return on Equity	Income statement and Balance sheet
Technical factors	TF-20DM	20-days Moving Average	Market
	TF-30DM	30-days Moving Average	Market
	TF-BOL	Bollinger bands	Market
	TF-MA	Price Moving Average Signal	Market
	TF-PP	Price Pivot Points Signal	Market
Market factors	MF-BP	Book-to-Price	Market and balance sheet
	MF-EP	Earnings-to-Price	Market and Income statement
	MF-SP	Sales-to-Price	Market and Income statement

Table A1: Factors used for establishing monthly scores for each stock from benchmark portfolio

We use three types of factors to establish for each stock. First group is fundamental factors. We use information from financial statements to score each stock. Because the statements are announced quarterly stays unchangeable for 3 months. This leads to high autocorrelation in signals as it is shown in column 4 of Table 1. Characteristics of these factors result in sufficiently low turnover for portfolios constructed on these factors.

Second group is technical factors. These factors are used by technical investment analysis for developing trading strategies. The source of information is only from stock price. This gives a characteristic of extremely actively changed indicators – practically they can be changed every millisecond. For the purpose of active portfolio management, we apply daily data for the prices of the stocks. Although the indicators are calculated on daily basis, we use only ones per month to rebalance the portfolio – this is done to keep comparability with other factors. Because of their extremely changeability the auto-regression of the factor forecasts is very low resulting in very high level of turnover – column 6 of Table 1.

Third group factors are combination between the first two. They are based on the market multipliers Price-to-Book, Price-Earning and Price-to-Sells ratios but in their reciprocal variant. These indicators combine the two sources of information – the fundamental (financial statements) and the price. This gives characteristic of modest activity and therefore – modest turnover.

APPENDIX 2:
How non-linear volatility impacts expected stock-specific risk

The standard GARCH (1,1) model for stock-specific risk at moment *t* takes the form of (A1):

$$\sigma_t^2 = \omega + \alpha * \epsilon_{t-1}^2 + \beta * \sigma_{t-1}^2 \tag{A1}$$

where:

ω – variance intercept parameter or unconditional sample variance (constant for all periods);
α – parameter governing the effect of recently realized unexpected volatility (shocks);
ε^2_{t-1} – realized unexpected volatility in the previous period;
β – parameter governing the effect of recently expected volatility;
σ^2_{t-1} – expected volatility in the previous period;

Similarly, the expected stock-specific risk at period t+1 is:

$$\sigma_{t+1}^2 = \omega + \alpha * \epsilon_t^2 + \beta * \sigma_t^2 \tag{A2}$$

Therefore, the difference in expectations from one period to another is:

$$\sigma_{t+1}^2 - \sigma_t^2 = \omega + \alpha * \epsilon_t^2 + \beta * \sigma_t^2 - \omega - \alpha * \epsilon_{t-1}^2 - \beta * \sigma_{t-1}^2 \tag{A3}$$

Rearranging (A3) it we get equation (A4) which is (5_ in the main text):

$$\Delta\sigma_{t+1}^2 = \alpha * (\epsilon_t^2 - \epsilon_{t-1}^2) + \beta * (\sigma_t^2 - \sigma_{t-1}^2) \tag{A4}$$