

# LONGEVITY RISK IN THE LIFE INSURANCE MARKET OF THE V4 COUNTRIES

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DOI: <https://doi.org/10.31410/EMAN.2018.213>

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**Abstract:** *Life insurance and pension systems have been exposed to new risks in recent decades. In addition to unfavorable demographic developments in the field of natality, they must also face the long-term phenomenon. The article presents an empirical analysis of the real data from the mortality tables of the V4 countries. The analysis shows that there is a significant prolongation of life in all four countries, which puts the threats of solvency. The paper also highlights the possibility of securitization of this risk.*

**Key words:** *Life insurance, longevity, risk transfer.*

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## 1. INTRODUCTION

Life in the modern world of the 21st century brings with it many risks. The basis of successful survival, whether individual or company, is to master their management. One of the key risk management tools is insurance. Insurance can be characterized as protection against financial loss caused by the acquisition of a certain risk.

In the last years, we could observe the new risk enters the life insurance business worldwide. It is the risk of the longevity of the insured. In the most developed countries has been observed the tendencies the people survive higher ages. The survey of the longevity brings [8] in chapter 4. It is shown here, the life expectancy has grown in the past 100 years. It has not only increased from 50 almost to 90 years in the most developed countries but as well in the least developed countries. The whole world average has increased to 80 years of the life expectancy at the birth. Therefore the governments, but also private insurers face to the financial risk that people will live on average longer than expected. The V4 countries are no exception.

In this paper, we briefly explain the computation principles of the life insurance. We will also document that the population of the V4 countries also exhibits the characteristics of the longevity in the recent decades. It naturally increases the risks of the life insurance business. Therefore we mention as well some risk transfer tools that can be used to hedge the longevity risk.

## 2. COMPUTATION PRINCIPLES AND DATA

The premium calculations in the life insurance are based on two main principles (see for example [3], [4], [5] or [9]). The first principle is the principle of the financial equivalence and the second one is assigned as the principle of the feigned insurance stock. While the principle of the financial equivalence reflects the time value of the incomes and liabilities, the second principle reflects the risks of the insurance.

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The risk entering into the insurance contract is usually measured by the probability of the insurance claim. These probabilities are reported in the life tables. In the life tables are reported the following values for each age  $x$ :

- $l_x$  number of persons living in the age  $x$  (portion from the starting 100 000),
- $d_x$  number of persons that died in the age  $x$ ,
- $p_x$  the probability that the individual in age  $x$  survives the next year,
- $q_x$  the probability, that the individual in age  $x$  dies in this age,
- $e_x$  the life expectancy.

Based on these values, we can further define the probabilities:

- ${}_t p_x$  the probability that the individual in age  $x$  survives the next  $t$  years,
- ${}_t q_x$  the probability, that the individual in age  $x$  dies during the next  $t$  years.

Clearly  ${}_t q_x = 1 - {}_t p_x$  and  ${}_t p_x = p_x \cdot p_{x+1} \cdot p_{x+2} \cdot \dots \cdot p_{x+n-1}$ .

Denoting  $r$  the interest rate and  $v = \frac{1}{1+r}$  the corresponding discount factor, we get the present value of the  $n$ -years unit ordinary annuity:

$$a_{\overline{n}|} = v + v^2 + \dots + v^n.$$

Combining this ordinary annuity with the life tables, we can express the present value of the unit temporary annuity, which represents paying off the fixed annuity on the end of each year during the insured period, while the insured person lives. If the entering age of the insured is  $x$ , we obtain

$$a_{x:\overline{n}|} = p_x \cdot v + {}_2 p_x \cdot v^2 + \dots + {}_n p_x \cdot v^n. \quad (1)$$

In the case of the life annuity, which has no formal terminal date but is paid until the death of the insured, assigning the terminal age as  $\omega$ , we get

$$a_x = p_x \cdot v + {}_2 p_x \cdot v^2 + \dots + {}_{\omega-x} p_x \cdot v^{\omega-x}. \quad (2)$$

In order to make relevant mortality analysis, it is necessary to get reliable data. For our purposes, we took The Human mortality database (HMD). The HMD began in the year 2000 as a collaborative project involving research teams in the Department of Demography at the University of California, Berkeley (USA) and at the Max Planck Institute for Demographic Research (MPIDR) in Rostock (Germany). The database was launched in May 2002 after its first phase of development. It is freely accessible at <http://www.mortality.org> and provides a highly valuable source of mortality statistics.

The HMD contains original calculations of death rates and life tables for national populations (countries or areas), as well as the input data used in constructing those tables. It includes life tables provided by single years of age up to 109, with an open terminal age for 110+.

### 3. LONGEVITY RISK IN THE V4 COUNTRIES

One of the most important functions of the life insurance is the protection against the risks like older age, disability and similar. The insurance policy is very frequently whole life investment, consisting of two periods – long period of paying the premiums and the second period of receiving the benefits. That means the policy is long lasting and its stability and the solvency

of the insurer strongly depends on the current mortality. It is clear, that the demographic parameters of the insurance can dramatically change during the insurance period, but the conditions must be stated in the beginning of the insurance business.

One of such risks associated with changes in mortality is the risk of the longevity. The longevity risk can be in general defined as the risk of populations living longer than expected. Longevity is the result of a complex interactions of various factors such as increased prosperity, changes in lifestyle, better education and progress in disease diagnostics and medical treatment, to mention a few.

As well the insurance business in the V4 countries faces with the problem of the longevity. The life expectancy at the birth has grown substantially in the past 100 years. This development is graphically illustrated in figure 1. There we can observe the increase of the life expectancy for both – men and women.

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If we consider the age-at-death to be a random variable  $X$ , we can describe the mortality by characteristics of this random variable. It is clear, that  $X$  is non-negative continuous random variable. It is fully characterized by its cumulative distribution function  $F(t)$  correspond to the probability  $P(X < t)$ , that the person dies before reaching the age  $t$ . The distribution of  $X$  can be also described by the survival function  $S(t) = 1 - F(t)$ , which is more useful to illustrate the longevity, as it represents the probability, that individual will live longer than  $t$ .

The survival functions for the newborn in the V4 countries are depicted in figure 2. On all pictures is easily observable the significant move of the survival function to the right among the years 1950 and 2014 (resp. 1958 and 2014 for Poland). It means increasing the chances the people to live longer.

All the graphs shows, that government, private companies and individuals all potentially face the longevity-born financial risk. Tables 1 — 8 illustrate the influence of that risk on the present values of the life annuities, stated by the formula (2), at fixed interest rate of 2%. These tables contain the present values of the unit life annuities for insured persons in three different ages in selected years from 1950 till 2014. The values demonstrate the increasing present discounted values of the liabilities of the defined-benefit pension plans, insurance companies that offer life annuities, and governments that sponsor old-age social security systems, as they would have to pay benefits longer than anticipated.

<i>Age</i>	<i>Year</i>							
	<i>1950</i>	<i>1960</i>	<i>1970</i>	<i>1980</i>	<i>1990</i>	<i>2000</i>	<i>2010</i>	<i>2014</i>
60	13.3751	14.4068	14.1513	14.3059	14.9514	16.3274	17.4537	17.9802
62	12.3070	13.3144	13.0757	13.2310	13.8526	15.2293	16.3841	16.9633
75	10.7644	11.6769	11.4529	11.5950	12.2474	13.5685	14.7331	15.2756

Table 1: Present values of the unit life annuities in selected years in the period from 1950 till 2014 for different aged women.in Czech republic. Supposed annual interest rate is 2 %.  
(Source: own calculations.)

<i>Age</i>	<i>Year</i>							
	<i>1950</i>	<i>1960</i>	<i>1970</i>	<i>1980</i>	<i>1990</i>	<i>2000</i>	<i>2010</i>	<i>2014</i>
60	11.8176	12.0786	11.3032	11.4610	11.6703	13.3401	14.5964	15.0987
62	10.9219	11.1488	10.3481	10.5473	10.7709	12.3966	13.6285	14.1150
65	9.5983	9.8381	8.9987	9.1552	9.4946	11.0025	12.2590	12.6765

Table 2: Present values of the unit life annuities in selected years in the period from 1950 till 2014 for different aged men.in Czech republic. Supposed annual interest rate is 2 %. (Source: own calculations.)

<i>Age</i>	<i>Year</i>							
	<i>1950</i>	<i>1960</i>	<i>1970</i>	<i>1980</i>	<i>1990</i>	<i>2000</i>	<i>2010</i>	<i>2014</i>
60	13.4865	13.8839	14.3151	14.3815	14.8974	15.7928	16.6597	16.9633
62	12.4820	12.8126	13.2258	13.3519	13.8673	14.7554	15.6534	15.9599
75	10.9959	11.1889	11.5927	11.7762	12.3179	13.1917	14.1025	14.4401

Table 3: Present values of the unit life annuities in selected years in the period from 1950 till 2014 for different aged women.in Hungary. Supposed annual interest rate is 2 %. (Source: own calculations.)

<i>Age</i>	<i>Year</i>							
	<i>1950</i>	<i>1960</i>	<i>1970</i>	<i>1980</i>	<i>1990</i>	<i>2000</i>	<i>2010</i>	<i>2014</i>
60	12.4255	12.4582	12.1261	11.6872	11.7700	12.4068	13.2344	13.7245
62	11.5178	11.4914	11.1515	10.7735	10.9635	11.5860	12.4213	12.8779
65	10.1762	10.0478	9.7534	9.4755	9.8066	10.4133	11.1919	11.6465

Table 4: Present values of the unit life annuities in selected years in the period from 1950 till 2014 for different aged men.in Hungary. Supposed annual interest rate is 2 %. (Source: own calculations.)

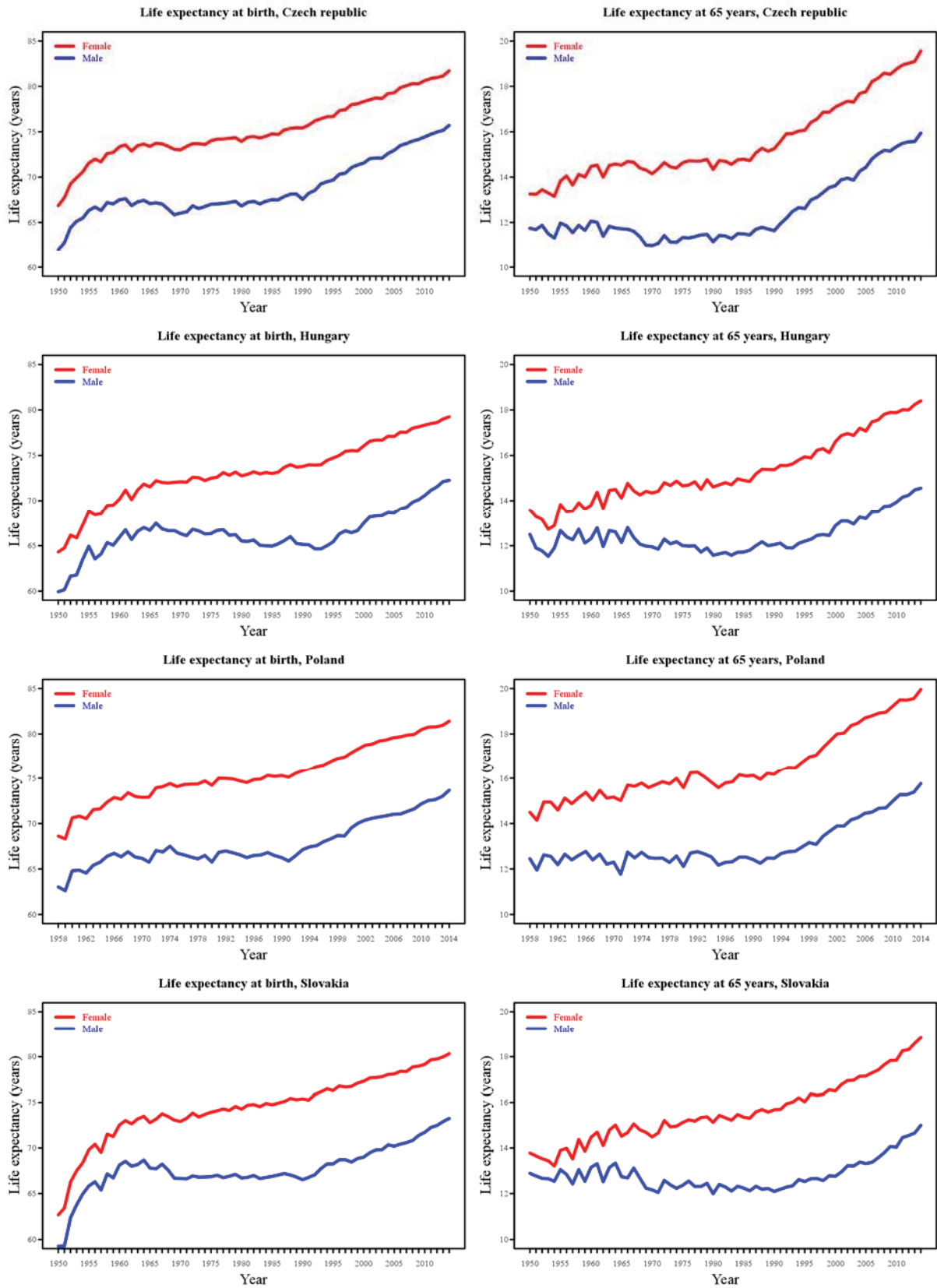


Figure 1: The life expectancy at birth (left) and life expectancy after at age 65 years (right) in the period from 1950 till 2014 within the V4 countries. (Source: Own processing, data [7])

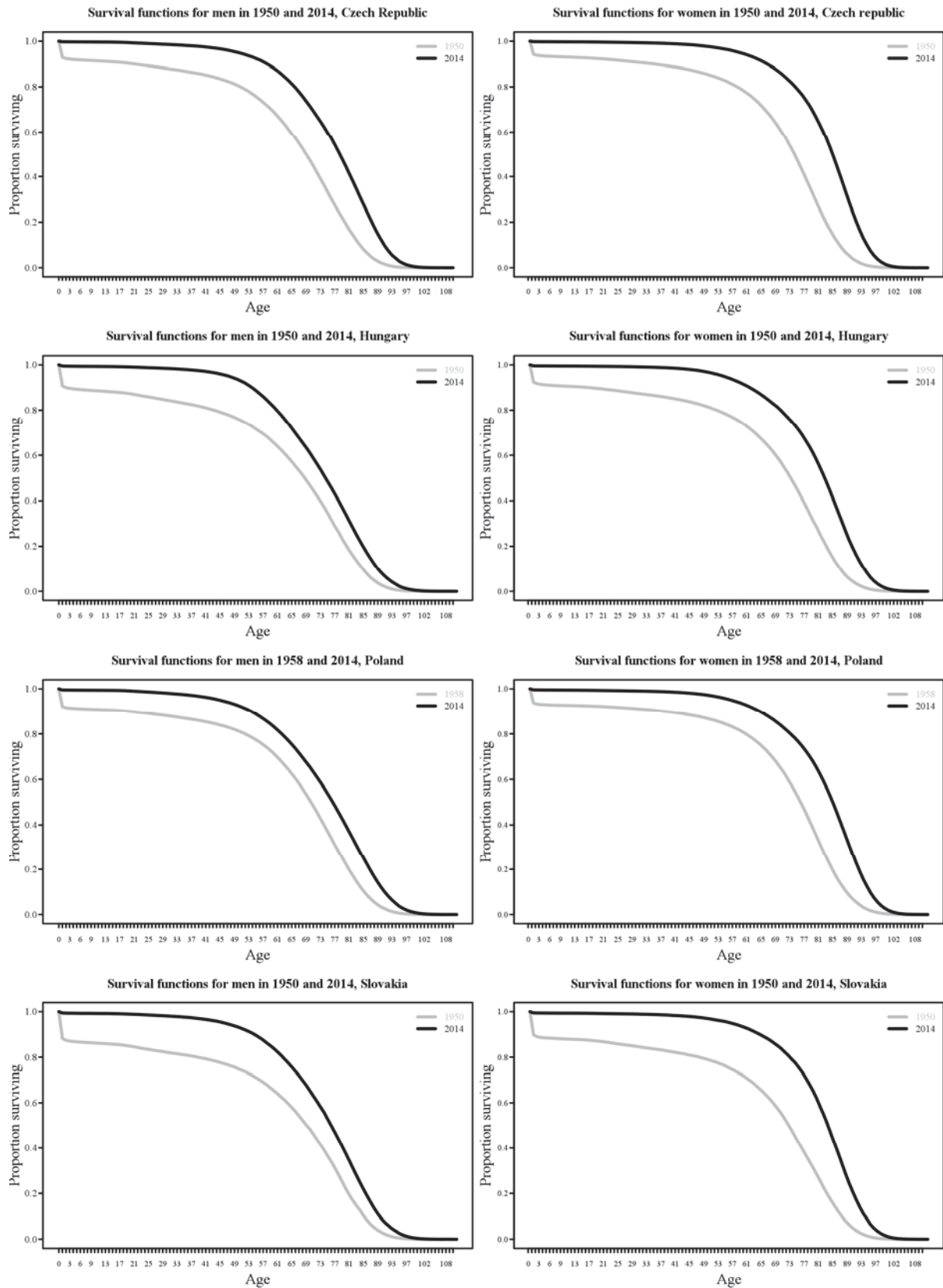


Figure 2: Graphs of the survival functions for the newborn men (left column) and women (right column) in the V4 countries. Comparison of the years 1950 and 2014. (Source: Own processing, data [7])

<i>Age</i>	<i>Year</i>							
	<i>1950</i>	<i>1960</i>	<i>1970</i>	<i>1980</i>	<i>1990</i>	<i>2000</i>	<i>2010</i>	<i>2014</i>
60	Na data	14.6471	14.8905	15.1656	15.4838	16.4565	17.6275	18.0747
62	No data	13.5842	13.8011	14.1099	14.4336	15.3851	16.5977	17.0595
65	No data	12.0036	12.1779	12.4897	12.8537	13.7689	15.0259	15.5015

Table 5: Present values of the unit life annuities in selected years in the period from 1950 till 2014 for different aged women.in Poland. Supposed annual interest rate is 2 %. (Source: own calculations.)

<i>Age</i>	<i>Year</i>							
	<i>1950</i>	<i>1960</i>	<i>1970</i>	<i>1980</i>	<i>1990</i>	<i>2000</i>	<i>2010</i>	<i>2014</i>
60	No data	12.5508	12.3674	12.1860	12.1717	13.0263	14.1706	14.7259
62	No data	11.6043	11.4027	11.2932	11.3416	12.1376	13.2779	13.8294
65	No data	10.2471	10.0114	9.8730	10.0794	10.8346	11.9574	12.5004

Table 6: Present values of the unit life annuities in selected years in the period from 1950 till 2014 for different aged men.in Poland. Supposed annual interest rate is 2 %. (Source: own calculations.)

<i>Age</i>	<i>Year</i>							
	<i>1950</i>	<i>1960</i>	<i>1970</i>	<i>1980</i>	<i>1990</i>	<i>2000</i>	<i>2010</i>	<i>2014</i>
60	13.6279	14.3760	14.4331	14.8298	15.2316	15.8616	16.8140	17.4552
62	12.6067	13.2875	13.3200	13.7611	13.8673	14.1349	15.7506	16.3986
65	11.1343	11.6803	11.7049	12.1792	12.5513	13.1491	14.1038	14.7820

Table 7: Present values of the unit life annuities in selected years in the period from 1950 till 2014 for different aged women.in Slovakia. Supposed annual interest rate is 2 %. (Source: own calculations.)

<i>Age</i>	<i>Year</i>							
	<i>1950</i>	<i>1960</i>	<i>1970</i>	<i>1980</i>	<i>1990</i>	<i>2000</i>	<i>2010</i>	<i>2014</i>
60	12.8275	13.1081	12.2681	12.1472	11.8484	12.4640	13.5262	14.2664
62	11.9001	12.1366	11.3167	11.2150	11.0410	11.6153	12.6131	13.3464
65	10.4826	10.0114	9.9140	9.7848	9.8336	10.3261	11.2934	11.9893

Table 8: Present values of the unit life annuities in selected years in the period from 1950 till 2014 for different aged men.in Slovakia. Supposed annual interest rate is 2 %. (Source: own calculations.)

#### 4. THE RISK TRANSFER

Life expectancy has been increasing over the last decades. It is positive people live longer, but on the other hand it also means they have to reconsider their savings in order to keep a satisfactory standard of living. How we can see from the figure 1, the life expectancy has increased in all V4 countries up to 15—18 years in the last half century.

- Shifting of the retirement age is the most common approach to solving the problem in public pension systems, applied by several governments. However, this approach has several weaknesses: The retirement age cannot be shifted unlimitedly.
- It is inapplicable to commercial insurance companies that can not unilaterally change the terms of the insurance contract.
- It is a denial of the underlying insurance principle, as it the risk is transferred from the insurer back to the insured.

Hedging of the longevity risk is highly desirable, but there has historically been limited appetite from reinsurers and no credible capital-market solution, [11] literally say: "Traditionally, reinsurance companies have provided capital to life insurance companies and pension funds seeking hedging opportunities. However, longevity risk is so specific that reinsurance companies have so far been reluctant to undertake business unless it was part of an existing client relationship."

The alternative risk transfer methods are modern techniques of the insurance industry and pension systems which are more appropriate in today's world than the classical cession of insurance risks as, for example in classical reinsurance. One of the typical solutions of the alternative risk transfer is securitization. We can characterize securitization as the process of taking an illiquid asset, or group of assets, and through financial engineering, transforming them into a security. In order to transfer insurance risk to the capital markets, the insurance and reinsurance companies use the high-yield debt instrument known as the insurance-linked securities (ILS). A large majority of the ILS market is composed of property catastrophe bonds (cat bonds), which are typically used as an alternative to buying traditional catastrophe reinsurance. Other types of ILS include mortality bonds or longevity bonds.

The payout on longevity bonds depends on the longevity experience of a given population, so that the payment is related to the number of survivors in the population. Longevity bonds (LBs for short) are instruments, whose payoffs  $f(t, S(t))$  are linked to the realized mortality of an underlying reference population, represented by a survivor index,  $S(t)$ . This index represents the proportion of initial population alive at some future time.

Longevity bonds can take a variety of forms, depending on the type of bond, survivor index chosen, specification of the payment function  $f(t, S(t, x))$ , maturity, credit risks involved, position to be hedged, and institution and portfolio type (e.g., life insurance or annuity contracts). The following list is based on [1]:

- Standard LBs: these are coupon-bearing bonds, whose coupon payments are linked to a survivor index and fall over time in line with the index. A simple example of a payment function could be  $f(t, S(t, x)) = kS(t, x)$ , for some  $k > 0$ .
- Inverse LBs: unlike standard LBs, the coupon payments of these are inversely related to a survivor index, rising over time instead of falling. In terms of the previous example, a payment function for inverse an LB might be of the form  $f(t, S(t, x)) = k(1 - S(t, x))$ ,  $k > 0$ .



- Longevity Zeros: these are zero coupon bonds, whose principal payments are functions of a survivor index. Longevity zeros could serve as building blocks for more complicated securities.
- Principal-at-risk LBs: these are longevity bonds, whose coupon payments (fixed or floating) are not affected by mortality development, but whose principal payment is linked to a survivor index.
- Survivor bonds: unlike standard longevity bonds, these have no specified maturity but continue to pay coupons as long as any individual belonging to the reference population is still alive: that is, the bonds have a stochastic maturity equal to the time of death of the last member of the reference population. There is no principal payment.

## 5. CONCLUSION

As we have seen, the risk of longevity is a complex and difficult predictable risk on the life insurance market. This complexity is based on its specificities compared to other insurance risks, in particular, the sensitivity to trends, geographic variability, the extremely long maturity of such insurances, and their potential correlation with many other risks. However, the longevity risk is far from being a concern for the insurance industry alone. It is indeed at the core of an open discussion for politicians, economists and strategists, who have to determine the "effective" pension scheme. The potential impacts of longevity risk at various levels of the global economy and society make a better management of this risk one of the key challenges of the coming decade.

## ACKNOWLEDGEMENT

This work has been supported by the grant of the Slovak Ministry of Education Nr. KEGA 041ŽU-4/2017

## REFERENCES

- [1] Blake, D., Cairns, a.,J., G., Dowd, K. (2006).: *Living with mortality: Longevity bonds and other mortality-linked securities*. British Actuarial Journal 12 (2006), pp. 153-228.
- [2] Blake, D., Cairns, A. J. G., Dowd, K., & MacMinn, R. (2006). *Longevity Bonds: Financial Engineering, Valuation, and Hedging*. The Journal of Risk and Insurance, 73(4), pp.647–672.
- [3] Cipra, T. (1994). *Insurance mathematics in the practice* (in Czech). Praha: Nakladatelství HZ s.r.o.
- [4] Cipra, T. (1996). *Pension insurance and its computational aspects* (in Czech). Praha: Nakladatelství HZ s.r.o.
- [5] Cipra, T. (2006). *Insurance mathematics. Theory and Practice* (in Czech). Praha: EKOPRESS, s.r.o.
- [6] Favero, C.A. and Giacoletti, M. (2011). *Progress in medicine, limits to life and forecasting mortality*. Milano, Italy: CEPR, NBER and Università Bocconi (Working Paper No. 406)
- [7] Human Mortality Database. University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany). Available at [www.mortality.org](http://www.mortality.org) or [www.humanmortality.de](http://www.humanmortality.de) (data downloaded on January, 20, 2018).
- [8] International Monetary Fund (2012). *Global Financial Stability Report. The Quest for Lasting Stability*. Global financial stability report, Washington, D.C..
- [9] Mehr, R. I., Cammack, E., & Rose, T. (1985). *Principles of Insurance* (8 Sub edition). Homewood, Ill: Richard D Irwin.

- [10] OECD. (2015). *Health at a Glance 2015*. OECD Publishing. Retrieved June 21, 2016, from [https://doi.org/10.1787/health\\_glance-2015-en](https://doi.org/10.1787/health_glance-2015-en). Retrieved November, 21, 2016.
- [11] Thomsen, J., & Andersen, J. V. (2007). *Longevity Bonds - a Financial Market Instrument to Manage Longevity Risk*. *Monetary Review*, 4th Quarter, pp. 29- 44.